

Diversity-Multiplexing Tradeoff in the Multiaccess Relay Channel with Finite Block Length

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Abstract

The Dynamic Decode-and-Forward (DDF) protocol and the Hybrid DDF and Amplified-and-Forward (HDAF) protocol for the multiple-access relay channel (MARC) with quasi static fading are evaluated using the Zheng-Tse diversity-multiplexing tradeoff (DMT). We assume that there are two users, one half-duplex relay, and a common destination, each equipped with single antenna. For the Rayleigh fading, the DDF protocol is well known and has been analyzed in terms of the DMT with *infinite* block length. By carefully dealing with properties specific to *finite* block length, we characterize the finite block length DMT which takes into account the fact that the event of decoding error at the relay causes the degradation in error performance when the block length is finite. Furthermore, we consider the situation where the destination does not have a priori knowledge of the relay decision time at which the relay switches from listening to transmitting. By introducing a decision rejection criterion such that the relay forwards message only when its decision is reliable, and the generalized likelihood ratio test (GLRT) rule at the destination that jointly decodes the relay decision time and the information message, our analysis show that the optimal DMT is achievable as if there is no decoding error at the relay and the relay decision time is known at the destination. Therefore, infinite block length and additional overhead for communicating the decision time are *not* needed for the DDF to achieve the optimal DMT. To further improve the DMT, we propose the HDAF protocol which take advantages of both the DDF and the Amplified-and-Forward protocols by judiciously choosing which protocol to use. Our result shows that the HDAF protocol outperforms the original DDF in the DMT perspective. Finally, a variant of the HDAF protocol with lower implementation complexity without sacrificing the DMT performance

is devised.

I. INTRODUCTION

In recent years, cooperative communication has received significant interest as a means of providing spatial diversity when time, frequency, antenna diversity are unavailable due to delay, bandwidth and terminal size constraints, respectively. Cooperative techniques provide diversity by enabling users to utilize one another's resources and has been extensively studied for the single source from outage probability analysis or diversity-multiplexing tradeoff (DMT) perspective [1], [2], [3].

Practical communication systems usually involve more than one user. One of the most typical models is the multiple-access channel (MAC). The capacity region of MAC is well known and the DMT is also developed in [4] for MAC. In [3], [5], cooperative diversity was extended to the multiple users cases. For most cooperative protocols, substantial coordination among the users are required, which may be impractical due to cost and complexity consideration. Alternatively, we consider the multiple-access relay channel (MARC), i.e. the MAC with a single shared relay and focus on the dynamic decode-and forward (DDF) protocol [3]. For DDF protocol, the relay does not decode until it is possible to successfully decode source information message. The relay then re-encodes the message and transmit it in the remaining coding interval. In this model we concern, the users need not be aware of the existence of the relay. All cost and complexity are placed in the relay and destination. Such an architecture may be suitable for infrastructure networks, where the relay and destination correspond the station having more resource (i.e. base station). Moreover, since a single relay is shared by multiple users in the MARC, the extra cost of adding the relay per user may thus be more acceptable. Finally, for further enhancement of DMT, we propose a hybrid protocol which combines DDF and multiple-access amplify-and-forward (MAF) [6], [7] protocol to improve the diversity gain at high multiplexing gain region.

A. Related Research

The MARC was first introduced in [8]. In MARC, the relay helps multiple sources simultaneously to reach a common destination. Information-theoretic treatment of the MARC has focused on two aspects, namely, the capacity region and the DMT (the outage behavior of slow fading channel in the high signal-to-noise (SNR) regime [9]). The achievable rate for the MARC

has been proposed in [10], [11], [12]. However, the capacity region of general MARC remains unknown. The DMT for the half-duplex MARC with single antenna nodes is studied in [6], [7], [13], [2]. In [6], [7], it is shown that MAF protocol is DMT optimal for high multiplexing gains; however, this protocol remains to be suboptimal for low multiplexing gains compared with DDF strategy [13]. In addition, the extra overhead to communicate the channel realizations of source-relay links to destination is required for MAF. Another relaying strategy for the MARC is compress-forward (CF) [2]. In CF, the relay exploits Wyner-Ziv coding to compress its received signal and forward it to destination. It has been shown in [2] that CF also achieves the optimal DMT for high multiplexing gains but suffers from diversity loss for low multiplexing gains, moreover, CF costs much larger complexity.

The present paper focuses on the DDF protocol for the half-duplex, single relay, single antenna case due to its nice balance between complexity and good performance at low multiplexing gain. Moreover, we propose a hybrid DDF-MAF (HDAF) protocol to enhance the DMT by improving the poor performance of DDF at high multiplexing gain.

B. Summary of Results

Previous work, [13], [3] assume an infinite block length such that there is no decoding error at the relay and the number of relay decision moments is also assumed infinite. Inspired by [14] for the single user, single relay case, we analyze explicitly the achievable DMT with finite block length and finite relay decision moments for the MARC. Note the former is a special case of the MARC when there is only one user. Moreover, different from the proof of DMT achievability in [14], we do not separately average the probability of event at relay and destination over the random codebook since they are not fully independent, which will be made more clear in Section B-C. As [14], two issues are discussed in our model: 1) the effect of decoding errors at the relay. 2) the fact that the relay decision time is not generally known priori at designation. In order to tackle 1) a decision rejection criterion at the relay, such that the relay triggers transmission only when its decoding is reliable. For 2), the destination jointly decodes the relay decision time and the information messages based on the generalized likelihood ratio test (GLRT) rule. Our results show that in order to achieve the DMT, additional protocol overhead informing the destination about the relay decision time is not necessary and the loss of DMT due to decoding error at the relay can be avoided. Finally, we propose HDAF protocol which takes advantage of

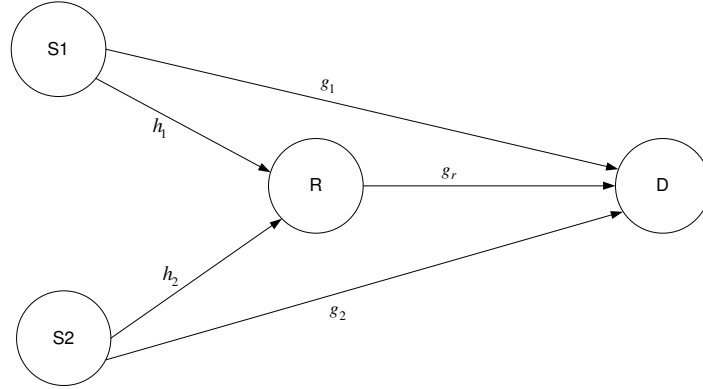


Fig. 1. MARC model

both DDF and MAF protocol. Our analysis shows that with the *finite* block length MT , HDAF protocol outperforms the original DDF protocol, especially at high multiplexing gain region and low multiplexing region when M is small. HDAF also has better DMT than MAF protocol when M is moderate large. In addition, without causing any loss in DMT perspective, a variant of HDAF with lower complexity by allowing the relay's transmission only after $M/2$ relay decision moments [15] is devised. Notice that the analysis with finite T will be much complicated since we have to deal with events which depends not only on channel realization (i.e. outage) but also on codebooks when applying random coding techniques.

In Section II, we introduce the system model and review relevant previous results. In Section III, a characterization of the DMT of the DDF protocol for MARC with finite block length is presented. In Section IV, we devise HDAF and its variant and characterize the DMT with finite block length. Section V gives the conclusion.

II. NOTATIONS AND DEFINITIONS

A. System Model

We Consider the two-user (S1 and S2) MARC model, a relay node (R) is assigned to assist the two multiple access users, (see Fig.1). The users are not allowed to help each other (due to practical limitations, for example). The relay node is constrained by the half-duplex assumption, i.e. the relay can not transmit and receive simultaneously. Each node equips with single antenna.

All wireless links are assumed to be frequency nonselective and block fading, where the channel coefficients are random but remain constant over the whole duration of a codeword and the channel coherence time is much larger than the allowed decoding delay. Let h_i, g_i, g_r denote the fading coefficients between the user i to relay, user i to destination, and relay to destination, respectively. The channel fading coefficients are i.i.d. $\mathcal{CN}(0,1)$ variables, corresponding to i.i.d. Rayleigh fading. Here we assume the perfect channel state information at receiver (CSIR). We adopt the slotted transmission where a codeword spans M slots and each slot consists of length T symbols. There are thus a total block length of MT . In decode-and-forward protocols, the block of length MT symbols is split into two phases. In the first phase, the relay receives the signal from the source until the end of a certain slot, referred as the decision time. Then the relay tries to decode the source message. In the second phase, based on the decoded message, the relay sends its codeword to the destination in the remaining block. For DDF protocol, the decision time depends on the channel coefficient and the received signal. For the first phase, the signal received by the relay is

$$y_{r,k} = \sum_i^2 h_i x_{i,k} + n_k, \quad k = 1, 2, \dots, \mathbf{m}T \quad (1)$$

Note the decision time at the end of slot \mathbf{m} is a random variable. The signal received by the destination is

$$y_{d,k} = \sum_i^2 g_i x_{i,k} + v_k, \quad k = 1, 2, \dots, \mathbf{m}T \quad (2)$$

During the second phase, the signal received by the destination is

$$y_{d,k} = \sum_i^2 g_i x_{i,k} + g_r x_{r,k} + w_k, \quad k = \mathbf{m}T + 1, \mathbf{m}T + 2, \dots, MT \quad (3)$$

We let $\mathbf{x}_i = [x_{i,1}, \dots, x_{i,MT}]^T$ denote the user i codeword with rate R_i bits per symbol. \mathbf{x}_r is similarly defined but only the last $(M - \mathbf{m})T$ symbols are transmitted. The same average power constraint P per symbol are imposed on each user and relay,

$$E[|x_{i,k}|^2] \leq P, \quad E[|x_{r,k}|^2] \leq P$$

where $E[\cdot]$ denotes expectation over all codewords. The noise at the relay and destination are independent Gaussian noise with variance of σ_n^2, σ_v^2 , denoted as $n_k \sim \mathcal{CN}(0, \sigma_n^2)$ and $w_k \sim \mathcal{CN}(0, \sigma_v^2)$ respectively. $\rho = \frac{P}{\sigma_v^2}$ and $\rho' = \frac{P}{\sigma_n^2}$ define the SNRs of the source-destination and the source-relay links, respectively. In our model, we consider the symmetric case where $R_1 = R_2 =$

$R/2$, the sum rate is R bits per symbol. As [13], the relay decision time is chosen to be the instant that the transmitted rate is within the achievable rate region at the end of \mathbf{m} slot, which satisfies

$$MTR_1 < m \log(1 + |h_1|^2 \rho') T \quad (4)$$

$$MTR_2 < m \log(1 + |h_2|^2 \rho') T \quad (5)$$

$$MTR < m \log(1 + (|h_1|^2 + |h_2|^2) \rho') T \quad (6)$$

\mathbf{m} is set to the minimum $m = 1, 2, \dots, M-1$ such that (4), (5), (6) hold, otherwise, $\mathbf{m} = M$ and the relay remains silent.

For later use, we denote the complement of an event A by \bar{A} , the transpose and Hermitian transpose of \mathbf{z} by \mathbf{z}^T , \mathbf{z}^H , respectively. $(x)^+ = x$ if $x > 0$, otherwise equal to zero. Let $\mathbf{x}_{i,n}^k$, and $\mathbf{x}_{r,n}^k$, denote the source transmit signal from time $nT + 1$ to kT and the relay transmit signal from time $nT + 1$ to kT respectively. $\mathbf{y}_{r,n}^k$, $\mathbf{x}_{d,n}^k$, are similarly defined.

B. Diversity-Multiplexing Tradeoff (DMT)

Our work use a lot the notion of DMT posed in [9]. We only provide definitions here. Consider a family of codes, such that the code has a rate of $R(\rho)$, corresponding to SNR ρ bits per channel use(BPCU) and error probability $P_E(\rho)$ The multiplexing gain r and the diversity gain defined as

$$r \triangleq \lim_{\rho \rightarrow \infty} \frac{R(\rho)}{\log \rho}, \quad d \triangleq - \lim_{\rho \rightarrow \infty} \frac{\log P_E(\rho)}{\log \rho} \quad (7)$$

and we can write as $P_E(\rho) \doteq \rho^{-d}$, where \doteq denotes the exponential equality. \leq and \geq are similarly defined. For the point to point multiple-input multiple-output(MIMO) channel with m transmit and n receive antennas, $r \leq \min(m, n)$, the optimal $d^*(r)$ is referred to as DMT.

III. DMT OF THE DDF PROTOCOL FOR MARC WITH FINITE BLOCK LENGTH

A. Effect of Finite Block Length

The DMT of DDF protocol for MARC with symmetric rate $R_1 = R_2 = \frac{r}{2} \log \rho$ has been shown in [13],

$$d_{DDF-MARC}^*(r) = \begin{cases} 2-r & \frac{1}{2} > r \geq 0 \\ 3(1-r) & \frac{2}{3} > r \geq \frac{1}{2} \\ 2\frac{1-r}{r} & 1 \geq r \geq \frac{2}{3} \end{cases} \quad (8)$$

However, to achieve the DMT in (8), a scheme with $M \rightarrow \infty$ possible decision times is necessary [13], furthermore, an infinite block length $T \rightarrow \infty$ is assumed such that there is no decoding error at the relay. For the practical code design, the code length is finite and the error event of decoding at the relay occurs even though the transmitted rate falls in the achievable rate region, i.e. (4), (5), (6) are satisfied. Forwarding the wrong source information message would significantly degrade the error performance at the destination. Thus the probability of relay decoding error dominates the error probability at the destination. The DMT analysis with finite M and T has been treated in [14] for the relay case, a special case of MARC model if there is only one user. In our MARC model, there are two users interfering with each other, thus the derivation of DMT is more involved, moreover, we take an approach different from [14] to averaging error probability over all the random codebook.

B. Characterization of DMT with Finite Block Length

In this section, the proof uses the machinery of [14], [13], [4], [3]. Therefore, we only provide a sketch of the previous results involved and focus on the novel parts. First, we find an upper bound on the DMT by letting $T \rightarrow \infty$, and assume the destination has the knowledge of \mathbf{m} . By relaxing the constraint that T is finite, we characterize the DMT using outage probability analysis, hence make it as an upper bound for the finite block length. This is established by the following theorem.

Theorem 1: The DMT of the two users, single-relay, single destination DDF scheme with decision times $m = 1, 2, \dots, M$ and finite slot length T is upper-bounded by

$$d_{out}(r) = \min_{1 \leq m \leq M} \{d_{m,R}(r) + d_{m,D}(r)\} \quad (9)$$

where $d_{m,R}(r)$ is defined in (40), and $d_{m,D}(r)$ is defined in (44)-(47).

Proof: We use similar techniques developed in [4], [14]. Let $P_{out}(r)$ denote the outage probability at the destination. $P_{out}^m(r)$ denote the outage probability at the destination for a given $\mathbf{m} = m$. The outage event O_D^m for a given $\mathbf{m} = m$ is defined as

$$O_D^m = \left\{ (g_1, g_2, g_r) : \left(O_{1,D}^m \cup O_{2,D}^m \cup O_{(1,2),D}^m \right) \right\} \quad (10)$$

where $O_{1,D}^m$, $O_{2,D}^m$, $O_{(1,2),D}^m$ are defined as

$$\begin{aligned} O_{1,D}^m = \{ (g_1, g_r) : mT \log(1 + |g_1|^2 \rho) \\ + (M - m)T \log(1 + (|g_1|^2 + |g_r|^2) \rho) \leq MTR_1 \} \end{aligned} \quad (11)$$

$O_{2,D}^m$ is similarly defined by replacing 1 with 2.

$$O_{(1,2),D}^m = \{(g_1, g_2, g_r) : mT \log(1 + (|g_1|^2 + |g_2|^2)\rho) + (M-m)T \log(1 + (|g_1|^2 + |g_2|^2 + |g_r|^2)\rho) \leq MTR\} \quad (12)$$

Then write

$$P_{out}(r) = \sum_{m=1}^M P(\mathbf{m} = m) P_{out}^m(r) \quad (13)$$

Since scaling SNR by a constant (ρ'/ρ) does not change the DMT. Define

$$\begin{aligned} P_{out}(r) &\doteq \rho^{-d_{out}(r)} \\ P_{out}^m(r) &\doteq \rho^{-d_{m,D}(r)}, \quad 1 \leq m \leq M \\ P(\mathbf{m} = m) &\doteq \rho^{-d_{m,R}(r)}, \quad 1 \leq m \leq M \end{aligned} \quad (14)$$

Then (9) clearly follows from (13). The proof of (14) is provided in Appendix A. It remains to show that it is an upper bound of finite block length case. From standard arguments based on Fano's inequality [4], it can be seen that $P_{out}^m(r)$ is indeed the best we can get, thus completes the proof. ■

Note in the proof above, the relay decision time \mathbf{m} is assumed priori known at the destination. The following theorem shows that this assumption is not necessary and the upper bound is achievable with finite T .

Theorem 2: The upper bound of Theorem 1 is achievable for finite-length T and no priori knowledge of decision time at the destination decoder.

Proof: In order to prove the achievability, we use standard random coding argument with bounded distance decoder [13] at the relay to overcome the effect of relay decoding error [14]. Owing to the introduction of the bounded distance decoder at the relay, the probability of relay's decision time at m slot and the probability of decoding error at destination given relay decision time m are not independent (the relay's decision time not only depends on the source-relay links but also on the codewords). Therefore, different from [14] where these two terms are *separately* averaged over the random ensemble, we take an approach to directly averaging the resulting error probability at the destination over the random ensemble.

Codebook Generation: For given $M, T, R_1 = R_2 = \frac{r}{2} \log \rho$, according to i.i.d components $\mathcal{CN}(0, P)$, independently generate $\rho^{\frac{rMT}{2}}$ codewords, $\mathcal{X}_i \subset \mathbb{C}^{MT}$, for each $i = 1, 2$ and $\mathcal{X}_r \subset \mathbb{C}^{MT}$

of cardinality ρ^{rMT} . We let $\mathbf{x}_i(w_i)$, $\mathbf{x}_r(w)$ denote the codewords in \mathcal{X}_i and in \mathcal{X}_r respectively, corresponding to the information message $w_i \in \{1, \dots, \rho^{\frac{rMT}{2}}\}$, $w \in \{1, \dots, \rho^{rMT}\}$.

Relay Decoding: From (4)-(6), we define the relay outage event at slot m as

$$\mathcal{O}_R^m = \left\{ (h_1, h_2) : \left(\mathcal{O}_{1,R}^m \cup \mathcal{O}_{2,R}^m \cup \mathcal{O}_{(1,2),R}^m \right) \right\} \quad (15)$$

and

$$\mathcal{O}_{1,R}^m \triangleq \left\{ h_1 : |h_1|^2 \leq \frac{\rho^{\frac{rM}{2}} - 1}{\rho'} \right\} \quad (16)$$

$\mathcal{O}_{1,R}^m$ is similarly defined.

$$\mathcal{O}_{(1,2),R}^m \triangleq \left\{ (h_1, h_2) : |h_1|^2 + |h_2|^2 \leq \frac{\rho^{\frac{rM}{2}} - 1}{\rho'} \right\} \quad (17)$$

Due to the finite block length MT , the relay may decode in error even $(h_1, h_2) \notin \mathcal{O}_R^m$. Then an incorrect codeword is sent to the destination, causing significant interference. Thereofre, similar to [16], [14] for the relay case, we introduce a bounded distance relay decoding decision function Ψ_δ defined as follows: for $m = 1, \dots, M-1$, define the regions $\mathcal{S}_m(w_1, w_2)$ consisting of all points $\mathbf{y} \in \mathbb{C}^{mT}$ for which (w_1, w_2) is the unique message enclosed in a sphere of squared radius $mT(1+\delta)\sigma_n^2$ centered at \mathbf{y} , i.e., $|\mathbf{y} - h_1\mathbf{x}_{1,0}^m(w_1) - h_2\mathbf{x}_{2,0}^m(w_2)|^2 \leq mT(1+\delta)\sigma_n^2$. If

- 1) $(h_1, h_2) \notin \mathcal{O}_R^m$,
- 2) $\mathbf{y}_{r,0}^m \in \mathcal{S}_m(\hat{w}_1, \hat{w}_2)$,

Then, $\Psi_\delta(\mathbf{y}_{r,0}^m, h_1, h_2)$ outputs the decoded message \hat{w} , where $\hat{w} = (\hat{w}_1, \hat{w}_2)$ and the relay starts to transmit the signal $\mathbf{x}_{r,m}^M(\hat{w})$ for the remaining part of the block. Otherwise, it waits for the next slot.

Destination Decoding : The destination is not aware of the relay decision time \mathbf{m} , hence it simultaneously detects the decision time and the information message according to the GLRT rule:

$$\{\hat{w}, \hat{m}\} = \arg \max_{w, m} p(\mathbf{y}_{d,0}^M | w, m) \quad (18)$$

where $p(\mathbf{y}_{d,0}^M | w, m)$ is the decoder likelihood function.

Error Probability Analysis : Let E denote the decoding error event at the destination and E_r denote the decoding error event at the relay. Follow the steps in [14], we have the following

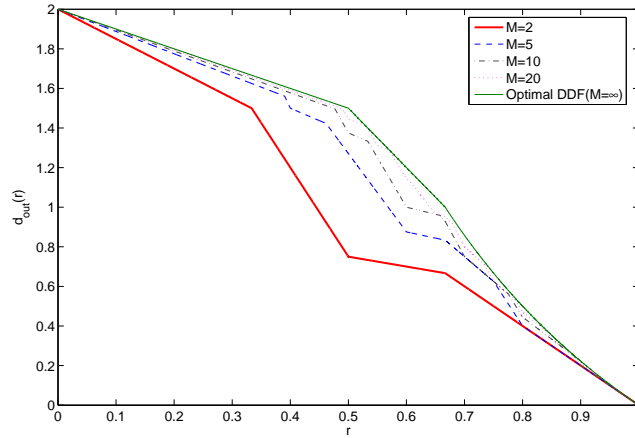


Fig. 2. The DMT of the DDF protocol for MARC with finite M

results,

$$P_E = \sum_{m=1}^M P(\mathbf{m} = m) P(E|\mathbf{m} = m) \quad (19)$$

$$\leq \sum_{m=1}^M P(\mathbf{m} = m) (P(E_r|\mathbf{m} = m) + P(E|\bar{E}_r, \mathbf{m} = m)) \quad (20)$$

For $m = 1, \dots, M-1$, let $\delta = \mu \log \rho$, we have

$$P(E_r|\mathbf{m} = m) \leq \rho^{-mT\mu} \quad (21)$$

Since $P(E|\bar{E}_r, \mathbf{m} = m) \geq \rho^{-d_{m,D}(r)}$, we can choose a sufficiently large finite μ to make the terms $P(E_r|\mathbf{m} = m)$ exponentially irrelevant in (20), i.e. since $d_{m,D}(r) < 3$, we can choose $\mu T > 3$. As for the other terms in (20), as mentioned earlier, they are not independent event since the relay's decision time not only depends on the source-relay links but also on the codewords. In Appendix B, unlike [14], by averaging these two terms together, we show the average probability of the term $P(\mathbf{m} = m)P(E_r|\mathbf{m} = m)$ using Gaussian random ensemble is exponentially upper bounded by $d_{out}(r)$, thus completes the proof. ■

It appears intractable to obtain a closed form of (9). In Fig.2, $d_{out}(r)$ are plotted for $M = 2, 5, 10, 20$. As M grows, $d_{out}(r)$ is seen to approach the optimal DMT in [13] where $M \rightarrow \infty$. Fig.2 also shows that even for a moderate value of M , $d_{out}(r)$ achieves the DMT close to optimal one.

IV. HYBRID AMPLIFIED-FORWARD AND DECODE-FORWARD PROTOCOL

A. The DMT of Hybrid Amplified-Forward and Decode-Forward protocol

It has been reported in [6] [7], the DMT of the MAF protocol is given by

$$d_{MAF}(r) = \begin{cases} 2 - \frac{3r}{2}, & 0 \leq r \leq \frac{2}{3} \\ 3(1 - r), & \frac{2}{3} \leq r \leq 1 \end{cases} \quad (22)$$

The optimal diversity gain for high multiplexing gain ($\frac{2}{3} \leq r \leq 1$) is achieved by MAF protocol. On the contrast, for the DDF strategy, the relay will only be able to during a small fraction of time slots, hence suffers form a loss compared to MAF. Combined with MAF protocol, the DMT may be further improved. This motivates us to propose a new hybrid strategy as follows:

- (1) For $\frac{2}{3} < r \leq 1$, the relay simply uses the MAF protocol.
- (2) For $\frac{1}{2} < r \leq \frac{2}{3}$, the relay simply uses the DDF protocol.
- (3) For $0 \leq r \leq \frac{1}{2}$, the relay dynamically decode the source messages before $M/2$ (including $M/2$) time slots ,where we assume M is even. If it can not successfully decode , then the MAF protocol is instead used after the time slot $M/2$.

The goal of the hybrid strategy aims to take advantages of both MAF and DDF protocol, we refer it as HDAF protocol. Note HDAF has the same setting as Section III (i.e. finite block length) when DDF is chosen to be used. We have the following theorem and Fig.2 showing that HDAF outperforms DDF especially at $\frac{2}{3} < r \leq 1$ and $0 \leq r \leq \frac{1}{2}$ when the number of time slots, namely M is small since HDAF can achieve the optimal DMT of DDF protocol with infinite M for $0 \leq r \leq \frac{1}{2}$ even a finite M time slots being used. Furthermore, HDAF has better diversity gain than MAF for $\frac{1}{2} < r \leq \frac{2}{3}$ when M is large enough since it will approach close to the DMT of DDF with infinite M .

Theorem 3: The DMT of HDAF protocol is

$$d^{HDAF}(r) = \begin{cases} 2 - r & 0 \leq r \leq \frac{1}{2} \\ d_{out}(r) & \frac{1}{2} < r \leq \frac{2}{3} \\ 3(1 - r) & \frac{2}{3} < r \leq 1 \end{cases} \quad (23)$$

Proof: To characterize the DMT of HDAF protocol $d_{HDAF}(r)$, for $\frac{2}{3} < r \leq 1$, the MAF protocol will be used and $d_{HDAF}(r) = d_{MAF}(r)$. The advantages of MAF for high multiplexing gain is preserved. For $\frac{1}{2} < r \leq \frac{2}{3}$, the DDF protocol will be used and $d_{HDAF}(r) = d_{out}(r)$ in (9),

hence this region entails no loss in terms of DMT compared to DDF protocol. For $0 \leq r \leq \frac{1}{2}$, from [13], we know $2 - r$ is indeed the upper bound for MARC, we will show that $d_{HDAF}(r) \geq 2 - r$, thus establish $d_{HDAF}(r) = 2 - r$. First consider the case where source message is decoded at the relay before $M/2$ time slots. In this case, since the upper bound of (9) is achievable by finite code length, we can similarly write $d_{HDAF}(r)$ as

$$d_{HDAF}(r) = \min_{1 \leq m \leq M/2} \left\{ d_{m,R}^{HDAF}(r) + d_{m,D}^{HDAF}(r) \right\} \quad (24)$$

It clear that $d_{m,R}^{HDAF}(r) = d_{m,R}(r)$, $d_{m,D}^{HDAF}(r) = d_{m,D}(r)$ for $1 \leq m \leq M/2$. From (44), the diversity gain at least $2 - r$ is obtained since $d_{m,D}^{HDAF}(r) = 2 - r$. However, this is not necessarily the case since the source message may be decoded only after $M/2$ time slots and the MAF protocol will be used instead. It remains to derive the DMT of HDAF protocol for the case where decoding moment $\mathbf{m} > M/2$. In [6], the error provability P_e using MAF protocol is shown to be upper-bounded by

$$P_e \leq P_{e_1} + P_{e_2} + P_{e_{(1,2)}} \quad (25)$$

where P_{e_I} , $I = 1, 2, (1, 2)$ represent the probability of the error event that user(s) I are detected in error. P_{e_I} 's are averaged over the ensemble of channel realizations, and thus leads to lower diversity gain $2 - \frac{3r}{2}$ for $0 \leq r \leq \frac{2}{3}$. However, for our case, the MAF protocol is used only when the relay can not decode the source message before $M/2$ time slots and the resulting diversity gain is expected to be larger. To deal with the HDAF case, from (48), the probability that the source message can not be decoded before $M/2$ time slots can be split into two events

$$\left\{ (h_1, h_2) \in O_R^{\frac{M}{2}} \right\} \cup \left\{ \{ (h_1, h_2) \notin O_R^{\frac{M}{2}} \}, \{ \mathbf{y}_{r,0}^{\frac{M}{2}} \notin \mathcal{U}_{\frac{M}{2}} \} \right\}$$

Then we need to upper bound the probability that MAF is used and decoding error occurs at the destination, defined as

$$\begin{aligned} P_{E,MAF} = & P \left(E, \left\{ (h_1, h_2) \in O_R^{\frac{M}{2}} \right\} \right) \\ & + P \left(E, \left\{ \{ (h_1, h_2) \notin O_R^{\frac{M}{2}} \}, \{ \mathbf{y}_{r,0}^{\frac{M}{2}} \notin \mathcal{U}_{\frac{M}{2}} \} \right\} \right) \end{aligned} \quad (26)$$

For the infinite block code length case, only the first term on the right hand side (26) need to be considered. Nevertheless, in our finite block code length case, the second term need to be taken into account which much complicates the analysis. We upper bound the first term here and leave the analysis of the second term in Appendix C.

Similar to (25), partition the first term on the right hand side of (26) into three mutually exclusive error events. To ease the notation, we denote them as P_{e_i} in this Section. Define $|h_i|^2 = \rho^{-\alpha_i}$, $|g_i|^2 = \rho^{-\beta_i}$, where $i = 1, 2, r$. From [3], the P_{e_1} conditioned on h_i, g_i is

$$P_{e_1|\alpha_1, \beta_1, \beta_r} \leq \rho^{-\frac{MT}{2}} [(\max\{2(1-\beta_1), 1-(\beta_r+\alpha_1)\})^+ - r] \quad (27)$$

and define the outage event as

$$O_{1,HDAF}^+ = \left\{ (\alpha_i, \beta_i) \in \left\{ \mathbb{R}^{5+} \cap \left(O_{1,R}^{\frac{M}{2}} \cup O_{2,R}^{\frac{M}{2}} \cup O_{(1,2),R}^{\frac{M}{2}} \right) \right\} : \right. \\ \left. (\max\{2(1-\beta_1), 1-(\beta_r+\alpha_1)\})^+ \leq r \right\} \quad (28)$$

From (16)-(17),

$$O_{i,R}^{\frac{M}{2}} = \left\{ \alpha_i : (1-\alpha_i)^+ \leq \frac{r}{2} \right\} \quad i = 1, 2$$

$$O_{(1,2),R}^{\frac{M}{2}} = \left\{ (\alpha_1, \alpha_2) : (1 - \min(\alpha_1, \alpha_2))^+ \leq r \right\}$$

Let $P_{e_i} \doteq \rho^{d_{P_{e_i}}(r)}$, by discussing cases of (28) for different events of $O_{1,R}^{\frac{M}{2}}$, it can be shown a diversity gain at least $2-r$ is obtained.

$$d_{P_{e_1}}(r) = \inf_{O_{1,HDAF}^+} \{ \alpha_1 + \alpha_2 + \beta_1 + \beta_2 + \beta_r \} \geq 2-r \quad (29)$$

$d_{P_{e_2}}(r)$ can be derived in identical manner. For $d_{P_{e_{(1,2)}}}(r)$, the DMT exponent conditioned on h_1, h_2 [6], is

$$d_{P_{e_{(1,2)}}|(h_1, h_2)}(r) \\ = \begin{cases} 2(1-r)^+ & \min\{\alpha_1, \alpha_2\} > (1-r)^+ \\ [3(1-r) - \min\{\alpha_1, \alpha_2\}]^+ & 0 \leq \min\{\alpha_1, \alpha_2\} \leq (1-r)^+ \end{cases} \quad (30)$$

Then

$$d_{P_{e_{(1,2)}}} = \inf_{(\alpha_1, \alpha_2) \in O_{1,R}^{\frac{M}{2}}} \left\{ \alpha_1 + \alpha_2 + d_{P_{e_{(1,2)}}|(h_1, h_2)}(r) \right\} > (2-r) \quad (31)$$

■

Theorem 3 shows that even for small M , HDAF can still achieve the optimal DMT of DDF protocol with infinite M when $0 \leq r \leq \frac{1}{2}$. The loss in DMT of finite M compared to infinite M case comes from the fact that the event of erroneous decoding at the destination when the relay starts transmission after $M/2$ time slots becomes the dominant error event, which will not occur in the HDAF strategy since MAF is used instead in that case.

B. Variant of HDAF protocol

Indicated in [15], allowing the relay node to start transmission at any time slots may result in higher complexity since this requires a very high-dimensional constellation to ensure the possibility of source message being uniquely decodable within a small code length. We can also prove the modified HDAF protocol that allowing the relay to transmit *only* after the $M/2$ time slots do not entail any loss in the DMT perspective.

Theorem 4: The modified HDAF protocol still achieves the DMT of Theorem 3.

Proof: Note DDF protocol is used only for $0 \leq r \leq \frac{2}{3}$. Replace (48) with 1 to upper bound $P(\mathbf{m} = m)$ for all $1 \leq m \leq M/2$, i.e. relay decodes before $M/2$ time slots. Following the remaining steps thereafter, we can obtain an upper bound $d_{m,D}(r)$. Observe (44)-(46), $d_{m,D}(r)$ is already equal to optimal value $2-r$, $3(1-r)$ for $0 \leq f \leq \frac{1}{2}$, $0 \leq f \leq \frac{5r-2}{2(3r-1)}$, respectively. Since $\frac{5r-2}{2(3r-1)} \geq \frac{1}{2}$ for $\frac{2}{3} \geq r \geq \frac{1}{2}$, decoding at $f \geq \frac{1}{2}$ is good enough, we conclude that the modification does not affect the DMT achieved by the HDAF protocol. ■

V. CONCLUSION

In this paper, we consider the design of cooperative protocol for MARC consisting half-duplex nodes. We analyze the DMT of DDF and proposed HDAD for MARC model with finite block length. The analysis captures the practical issue that the relay may decode erroneously even when the channel is not outage and the destination have no priori knowledge of the relay's decision moment. Our results show that additional protocol overhead informing the destination about the relay decision time is not necessary and the loss in DMT due to decoding error at the relay can be overcome. The difficulty of analysis comes from that we need to properly manipulate the events related to not only channel realization(i.e outage) but also codewords when applying random coding schemes with finite block length. We also propose HDAF protocol which shares both the advantages of DDF at low-medium multiplexing gain and MAF at high multiplexing gain region. It achieves higher(optimal) diversity gain than DDF protocol, particularly at high multiplexing gain and at low multiplexing gain when the number of time slots M is small. HDAF also outperforms MAF at low-medium multiplexing gain region when M is reasonable large enough. Finally, we have investigated the variant of HDAF with reduced complexity by allowing the relay to switch to transmission mode only after half of codeword and prove there is no entailing loss in terms of DMT compared to the original HDAF.

APPENDIX A
PROOF OF THEOREM 1

Since $d_{m,R}(r)$ is solely a function of the source-relay links and $d_{m,D}(r)$ is a function of relay-destination and source-destination links. These two terms can be analyzed separately.

A. Analysis of $d_{m,R}(r)$

We partition the event $\{\mathbf{m} = m\}$ into the set of events A_I^m , i.e. $\{\mathbf{m} = m\} = \bigcup_I A_I^m$, where I denotes any nonempty subset of $\{1, 2\}$. For notation convenience, we make two extra definitions that are only used in Appendix A and Appendix B, $\overline{O_{I,R}^0} = \phi$, $O_R^M = \phi$ where ϕ denotes the empty set. Then for $m = 1, \dots, M$, A_I^m are defined as follows

$$A_1^m = \left\{ (h_1, h_2) : O_{1,R}^{m-1} \cap \overline{O_R^m} \right\} \quad (32)$$

Event A_2^m is similarly defined with 1 replaced with 2.

$$A_{(1,2)}^m = \left\{ (h_1, h_2) : O_{(1,2),R}^{m-1} \cap \overline{O_R^m} \right\} \quad (33)$$

A_I^m denotes the event that source message can be decoded at slot m and the outage event $O_{I,R}^{m-1}$ occurs. Note

$$P(A_{I^*}^m) \leq P(\mathbf{m} = m) = P\left(\bigcup_I A_I^m\right) \leq \sum_I P(A_I^m) \doteq P(A_{I^*}^m) \quad (34)$$

where $I^* = \arg \min_I \lim_{\rho' \rightarrow \infty} \frac{-\log P(A_I^m)}{\log \rho'}$. Thus $P(\mathbf{m} = m) \doteq P(A_{I^*}^m)$. Since $|h_i|^2$ are exponentially distributed and $\rho \doteq \rho'$, we compute $P(A_I^m) \doteq \rho^{d_{A_I^m}(r)}$ as follows:

1) Computation of $d_{A_1^m}(r)$, for $M-1 \geq m \geq 2$: For $r \geq 0$, $M-1 \geq m \geq 2$

$$P(A_1^m) = \begin{cases} r > \frac{m}{M}, & d_{A_1^m}(r) = \infty \\ r \leq \frac{m}{M}, & d_{A_1^m}(r) = 1 - \frac{Mr}{2(m-1)} \end{cases} \quad (35)$$

Due to the symmetry, $d_{A_2^m}(r) = d_{A_1^m}(r)$ is clear.

2) Computation of $d_{A_{1,2}^m}(r)$, for $M-1 \geq m \geq 2$:

$$P(A_{1,2}^m) = \begin{cases} r > \frac{m}{M}, & d_{A_{1,2}^m}(r) = \infty \\ \frac{m}{M} \geq r > \frac{m-1}{M}, & d_{A_{1,2}^m}(r) = 0 \\ \frac{m-1}{M} \geq r, & d_{A_{1,2}^m}(r) = 2\left(1 - \frac{Mr}{m-1}\right) \end{cases} \quad (36)$$

3) *Computation of $d_{m=1,R}(r)$:*

$$P(\mathbf{m} = 1) = \begin{cases} r > \frac{m}{M}, & d_{m=1,R}(r) = \infty \\ \frac{m}{M} \geq r, & d_{m=1,R}(r) = 0 \end{cases} \quad (37)$$

4) *Computation of $d_{m=M,R}(r)$:*

$$P(A_1^M) = \begin{cases} r > \frac{2(M-1)}{M}, & d_{A_1^M}(r) = 0 \\ \frac{2(M-1)}{M} \geq r, & d_{A_1^M}(r) = 1 - \frac{Mr}{M-1} \end{cases} \quad (38)$$

$d_{A_2^M}(r) = d_{A_1^M}(r)$ is clear.

$$P(A_{1,2}^M) = \begin{cases} r > \frac{(M-1)}{M}, & d_{A_{1,2}^M}(r) = 0 \\ \frac{(M-1)}{M} \geq r, & d_{A_{1,2}^M}(r) = 2 \left(1 - \frac{Mr}{M-1}\right) \end{cases} \quad (39)$$

Collecting the results (35)-(39), overall, we obtain

$$d_{m,R}(r) = \begin{cases} 0 \leq r < \frac{2(m-1)}{3M}, & d_{m,R}(r) = 1 - \frac{Mr}{2(m-1)} \\ \frac{2(m-1)}{3M} \leq r < \frac{(m-1)}{M}, & d_{m,R}(r) = 2 \left(1 - \frac{Mr}{(m-1)}\right) \\ \frac{(m-1)}{M} \leq r \leq \frac{m}{M}, & d_{m,R}(r) = 0 \\ \frac{m}{M} < r \leq 1, & d_{m,R}(r) = \infty \end{cases} \quad (40)$$

B. Analysis of $d_{m,D}(r)$

Similar to (34), $P_{out}^m(r) \doteq P_{O_{r^*,D}}^m(r)$ the DMT $d_{O_{r^*,D}}^m(r)$ has been derived in [13], we provide results here, let $f = \frac{m}{M}$,

$$d_{O_{1,D}}^m(r) = \begin{cases} \frac{1}{2} > f \geq 0, & 2 - r \\ 1 - \frac{r}{2} > f \geq \frac{1}{2}, & 2 - \frac{r}{2(1-f)} \\ 1 \geq f \geq 1 - \frac{r}{2}, & \frac{(2-r)}{2f} \end{cases} \quad (41)$$

and

for $\frac{1}{3} > r \geq 0$,

$$d_{O_{(1,2),D}}^m(r) = \begin{cases} \frac{2}{3} > f \geq 0, & 3(1-r) \\ 1-r > f \geq \frac{2}{3}, & 3 - \frac{r}{1-f} \\ 1 \geq f \geq 1-r, & 2\frac{1-r}{f} \end{cases} \quad (42)$$

for $1 \geq r \geq \frac{1}{3}$,

$$d_{O_{(1,2),D}^m}(r) = \begin{cases} \frac{2}{3} > f \geq 0, & 3(1-r) \\ 1 \geq f \geq \frac{2}{3}, & 2\frac{1-r}{f} \end{cases} \quad (43)$$

Note $d_{m,D}(r) = \min \{d_{O_{(1,2),D}^m}(r), d_{O_{1,D}^m}(r), d_{O_{2,D}^m}(r)\}$. By some manipulation, we obtain,

for $\frac{1}{2} > r \geq 0$,

$$d_{m,D}(r) = \begin{cases} \frac{1}{2} > f \geq 0, & 2-r \\ 1 - \frac{r}{2} > f \geq \frac{1}{2}, & 2 - \frac{r}{2(1-f)} \\ 1 \geq f \geq 1 - \frac{r}{2}, & \frac{2-r}{2f} \end{cases} \quad (44)$$

for $\frac{16}{25} > r \geq \frac{1}{2}$,

$$d_{m,D}(r) = \begin{cases} \frac{5r-2}{2(3r-1)} > f \geq 0, & 3(1-r) \\ 1 - \frac{r}{2} > f \geq \frac{5r-2}{2(3r-1)}, & 2 - \frac{r}{2(1-f)} \\ 1 \geq f \geq 1 - \frac{r}{2}, & \frac{2-r}{2f} \end{cases} \quad (45)$$

for $\frac{2}{3} > r \geq \frac{16}{25}$,

$$d_{m,D}(r) = \begin{cases} \frac{5r-2}{2(3r-1)} > f \geq 0, & 3(1-r) \\ \frac{2 - \frac{5r}{4} - \sqrt{r(\frac{25r}{16} - 1)}}{2} > f \geq \frac{5r-2}{2(3r-1)}, & 2 - \frac{r}{2(1-f)} \\ \frac{2 - \frac{5r}{4} + \sqrt{r(\frac{25r}{16} - 1)}}{2} > f \geq \frac{2 - \frac{5r}{4} - \sqrt{r(\frac{25r}{16} - 1)}}{2}, & \frac{2(1-r)}{f} \\ 1 - \frac{r}{2} > f \geq \frac{2 - \frac{5r}{4} + \sqrt{r(\frac{25r}{16} - 1)}}{2}, & 2 - \frac{r}{2(1-f)} \\ 1 \geq f \geq 1 - \frac{r}{2}, & \frac{2-r}{2f} \end{cases} \quad (46)$$

for $1 \geq r \geq \frac{2}{3}$,

$$d_{m,D}(r) = \begin{cases} \frac{2}{3} > f \geq 0, & 3(1-r) \\ 1 \geq f \geq \frac{2}{3}, & \frac{2(1-r)}{f} \end{cases} \quad (47)$$

APPENDIX B

PROOF OF THEOREM 2

We now turn to the analysis of the term $P(\mathbf{m} = m)P(E_r|\mathbf{m} = m)$ in (20). First, we upper bound these two terms by extending the techniques in [14] to our MARC model. Then, we average the product of these two terms over the Gaussian random ensemble

A. Analysis of $P(\mathbf{m} = m)$

Let $\mathcal{U}_m = \bigcup_{w=1}^{\rho^{rMT}} \mathcal{S}_m(w)$, where $w = (w_1, w_2)$. Averaged with respect to the random coding ensemble, we may choose without loss of generality $w_1 = 1, w_2 = 1$ as the reference transmitted message. Let $\Delta \mathbf{x}_{i,0}^m(w_i) = \mathbf{x}_{i,0}^m(w_i) - \mathbf{x}_{i,0}^m(1)$, as [14], with h replaced by (h_1, h_2) , for $1 \leq m \leq M$, we have the following results

$$\begin{aligned} P(\mathbf{m} = m) &\leq P\left(\{(h_1, h_2) \notin O_R^{m-1}\}, \{\mathbf{y}_{r,0}^{m-1} \notin \mathcal{U}_{m-1}\}\right) \\ &\quad + P\left(\{(h_1, h_2) \in O_R^{m-1}\}, \{(h_1, h_2) \notin O_R^m\}\right) \end{aligned} \quad (48)$$

where the definition $\overline{O_R^0} = \phi$, $O_R^M = \phi$ are again used.

For $1 < m < M$, given a (h_1, h_2) and a codebook,

$$\begin{aligned} P(\{\mathbf{y}_{r,0}^m \notin \mathcal{U}_m\}) &\leq (1 + \delta)^{mT} e^{-mT\delta} \\ &\quad + \sum_{w \neq (1,1)} I_d\{|h_1 \Delta \mathbf{x}_{1,0}^m(w_1) + h_2 \Delta \mathbf{x}_{2,0}^m(w_2)|^2 \leq 4mT(1 + \delta)\sigma_n^2\} \end{aligned} \quad (49)$$

where $I_d(\cdot)$ is the indicator function. Different from [14], we do *not* average over the random coding ensemble here from (49). Instead, we use the following inequality

$$\begin{aligned} I_d(|h_1 \Delta \mathbf{x}_{1,0}^m(w_1) + h_2 \Delta \mathbf{x}_{2,0}^m(w_2)|^2 \leq 4mT(1 + \delta)\sigma_n^2) \\ \leq e^1 \exp\left\{-\frac{|h_1 \Delta \mathbf{x}_{1,0}^m(w_1) + h_2 \Delta \mathbf{x}_{2,0}^m(w_2)|^2}{4mT(1 + \delta)\sigma_n^2}\right\} \end{aligned} \quad (50)$$

Then we turn to upper bound the term $P(E|\overline{E}_r, \mathbf{m} = m)$

B. Analysis of $P(E|\overline{E}_r, \mathbf{m} = m)$

We consider the GLRT decoder at the destination. This decoder has no knowledge of decision time \mathbf{m} . Thus the receiver has to decode both the decision time and source information message.

The $P(E|\bar{E}_r, \mathbf{m} = m)$ is upper bounded by

$$\begin{aligned} P(E|\bar{E}_r, \mathbf{m} = m) &\leq P(\{(g_1, g_2, g_r) \in O_D^m\}) \\ &+ P(E, \{(g_1, g_2, g_r) \notin O_D^m\}|\bar{E}_r, \mathbf{m} = m) \end{aligned} \quad (51)$$

and similar to [14], we have

$$\begin{aligned} &P(\{(1, 1) \rightarrow \tilde{w}\}|\bar{E}_r, \mathbf{m} = m) \\ &\leq \sum_{m'=1}^M P(p(\mathbf{y}_{d,0}^M|1, m) \leq p(\mathbf{y}_{d,0}^M|\tilde{w}, m')|\bar{E}_r, \mathbf{m} = m) \end{aligned} \quad (52)$$

and for $m' \geq m$, (the case for $m' < m$ can be derived by interchanging the role of m' and m , thus omitted here), the term inside the sum (52) can be upper bounded by

$$\begin{aligned} &P(p(\mathbf{y}_{d,0}^M|1, m) \leq p(\mathbf{y}_{d,0}^M|\tilde{w}, m')|\bar{E}_r, \mathbf{m} = m) \leq e^{-|\mathbf{z}_{m'}|^2/(4\sigma_v^2)} \\ &\mathbf{z}_{m'} \triangleq \begin{bmatrix} g_1 \Delta \mathbf{x}_{1,0}^m(\tilde{w}_1) + g_2 \Delta \mathbf{x}_{2,0}^m(\tilde{w}_2) \\ g_1 \Delta \mathbf{x}_{1,m}^{m'}(\tilde{w}_1) + g_2 \Delta \mathbf{x}_{2,m}^{m'}(\tilde{w}_2) + g_r \mathbf{x}_{r,m}^{m'}(1, 1) \\ g_1 \Delta \mathbf{x}_{1,m'}^M(\tilde{w}_1) + g_2 \Delta \mathbf{x}_{2,m'}^M(\tilde{w}_2) + g_r \Delta \mathbf{x}_{r,m'}^M(\tilde{w}) \end{bmatrix} \end{aligned} \quad (53)$$

C. Averaged over the Gaussian random ensemble

We are ready to average the term $P(\mathbf{m} = m)P(E|\bar{E}_r, \mathbf{m} = m)$ over the Gaussian random ensemble. From (48),(51), for $1 \leq m \leq M$, it can be upper bounded by sum of the four terms discussed below, we will show that the four terms have exponents larger than or equal to $d_{m,D}(r) + d_{m,R}(r)$:

$$(1) P(\{(g_1, g_2, g_r) \in O_D^m\})P(\{(h_1, h_2) \in O_R^{m-1}\}, \{(h_1, h_2) \notin O_R^m\}):$$

It is clear this term has the exponent of $d_{m,D}(r) + d_{m,R}(r)$.

$$(2) P(E, \{(g_1, g_2, g_r) \notin O_D^m\}|\bar{E}_r, \mathbf{m} = m)P(\{(h_1, h_2) \notin O_R^{m-1}\}, \{\mathbf{y}_{r,0}^{m-1} \notin \mathcal{U}_{m-1}\}):$$

This term is more involved. For the case $m = 1$, the term (2) becomes to

zero. For $1 < m \leq M$, given h_i, g_i , from (48), (50), (52), (53), we have

$$P\left(\{\mathbf{y}_{r,0}^{m-1} \notin \mathcal{U}_{m-1}\}\right) \cdot P(\{(1,1) \rightarrow \tilde{w}\}|\bar{\mathbf{E}}_r, \mathbf{m} = m) \quad (54)$$

$$\leq \left[\sum_{w \neq (1,1)} e^1 \exp \left\{ -\frac{|h_1 \Delta \mathbf{x}_{1,0}^{m-1}(w_1) + h_2 \Delta \mathbf{x}_{2,0}^{m-1}(w_2)|^2}{(4(m-1)T(1+\delta)\sigma_n^2)} \right\} \right. \\ \left. + (1+\delta)^{(m-1)T} e^{-(m-1)T\delta} \right] \cdot \sum_{m'=1}^M e^{-|\mathbf{z}_{m'}|^2/(4\sigma_v^2)} \quad (55)$$

$$\leq \sum_{w \neq (1,1)} e^1 \exp \left\{ -\frac{|h_1 \Delta \mathbf{x}_{1,0}^{m-1}(w_1) + h_2 \Delta \mathbf{x}_{2,0}^{m-1}(w_2)|^2}{(4(m-1)T(1+\delta)\sigma_n^2)} \right\} \\ \cdot \sum_{m'=1}^M e^{-|\mathbf{z}_{m'}|^2/(4\sigma_v^2)} \quad (56)$$

$$\leq \sum_{w \neq (1,1)} \exp \left\{ -\frac{|h_1 \Delta \mathbf{x}_{1,0}^{m-1}(w_1) + h_2 \Delta \mathbf{x}_{2,0}^{m-1}(w_2)|^2}{(4(m-1)T(1+\delta)\sigma_n^2)} \right\} \\ \cdot \sum_{m'=1}^M e^{-|\mathbf{z}_{m'}|^2/(4(m-1)T(1+\delta)\sigma_v^2)} \quad (57)$$

$$= \sum_{w \neq \{(1,1), \tilde{w}\}} \exp \left\{ -\frac{|h_1 \Delta \mathbf{x}_{1,0}^{m-1}(w_1) + h_2 \Delta \mathbf{x}_{2,0}^{m-1}(w_2)|^2}{(4(m-1)T(1+\delta)\sigma_n^2)} \right\} \\ \cdot \sum_{m'=1}^M e^{-|\mathbf{z}_{m'}|^2/(4(m-1)T(1+\delta)\sigma_v^2)} + \sum_{m'=1}^M e^{-|\mathbf{z}'_{m'}|^2/(4(m-1)T(1+\delta)\sigma_v^2)} \quad (58)$$

where

$$\mathbf{z}'_{m'} \triangleq \left[\frac{\sigma_v}{\sigma_n} \left(h_1 \Delta \mathbf{x}_{1,0}^{m-1}(\tilde{w}_1) + h_2 \Delta \mathbf{x}_{2,0}^{m-1}(\tilde{w}_2) \right) \quad \mathbf{z}_{m'} \right]$$

Note by separating the term of \tilde{w} from the first sum in (58), then the product of the first two sum in (58) can now be averaged separately. We first consider the single term

$$e^{-|\mathbf{z}'_{m'}|^2/(4(m-1)T(1+\delta)\sigma_v^2)} \quad (59)$$

Let

$$\mathbf{X}_i = [\Delta x_{1,i}(\tilde{w}_1), \Delta x_{2,i}(\tilde{w}_2), x_{r,i}(1,1), \Delta x_{r,i}(\tilde{w})]^T, \quad 1 \leq i \leq MT$$

and

$$\mathbf{X} = [\mathbf{X}_1^T, \mathbf{X}_2^T, \dots, \mathbf{X}_{MT}^T]^T$$

It can be verified that $|\mathbf{z}'_m|^2 = \mathbf{X}^H \mathbf{R} \mathbf{X}$, where \mathbf{R} is a block diagonal matrix of the form $\mathbf{R} = \text{diag}(\mathbf{R}_1, \dots, \mathbf{R}_{MT})$.

For $1 \leq i \leq (m-1)T$,

$$\mathbf{R}_i = \mathbf{C}^H \mathbf{C}$$

where $h'_i = \frac{\sigma_v^2}{\sigma_n^2} h_i$ and

$$\mathbf{C} = \begin{bmatrix} h'_1 & h'_2 & 0 & 0 \\ g_1 & g_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

For $(m-1)T+1 \leq i \leq mT$

$$\mathbf{R}_i = \mathbf{g}_a^H * \mathbf{g}_a, \quad \mathbf{g}_a = [g_1, g_2, 0, 0]$$

For $mT+1 \leq i \leq m'T$

$$\mathbf{R}_i = \mathbf{g}_b^H * \mathbf{g}_b, \quad \mathbf{g}_b = [g_1, g_2, g_r, 0]$$

For $m'T+1 \leq i \leq MT$

$$\mathbf{R}_i = \mathbf{g}_c^H * \mathbf{g}_c, \quad \mathbf{g}_c = [g_1, g_2, 0, g_r]$$

Let $\mathbf{R}^s = \text{diag}(\mathbf{R}_1^s, \dots, \mathbf{R}_{MT}^s)$, $\mathbf{D} = \text{diag}(\mathbf{D}_1, \dots, \mathbf{D}_{MT})$ where \mathbf{R}_i^s denotes \mathbf{R}_i with the s -th row and the s -th column replaced by zero vector. (when $s=0$, nothing is changed). $\mathbf{D}_i = \text{diag}(2P, 2P, P, 2P)$.

\mathbf{C}^s is similarly defined. Averaging (59) over the Gaussian random ensemble, we have

$$\frac{1}{\det \left(\mathbf{I} + \frac{\mathbf{D} \mathbf{R}^s}{(4(m-1)T(1+\delta)\sigma_v^2)} \right)} \quad (60)$$

Let $k = \frac{1}{((m-1)T(1+\delta))}$, we have

(I) For $\{\tilde{w}_1 \neq 1, \tilde{w}_2 \neq 1\}$, $s = 0$,

$$\begin{aligned}
\frac{1}{\det\left(\mathbf{I} + \frac{k\mathbf{D}\mathbf{R}^s}{4}\right)} &= \frac{1}{\det\left(\mathbf{I} + \frac{k\rho\mathbf{C}^H\mathbf{C}}{2}\right)^{(m-1)T}} \\
&\cdot \frac{1}{\left[1 + \frac{k\rho(|g_1|^2 + |g_2|^2)}{2}\right]^T} \\
&\cdot \frac{1}{\left[1 + \frac{k\rho(2|g_1|^2 + 2|g_2|^2 + |g_r|^2)}{4}\right]^{(m'-m)T}} \\
&\cdot \frac{1}{\left[1 + \frac{k\rho(|g_1|^2 + |g_2|^2 + |g_r|^2)}{2}\right]^{(M-m')T}} \\
&\leq \frac{1}{[1 + \rho(|h_1|^2 + |h_2|^2 + |g_1|^2 + |g_2|^2)]^{(m-1)T}} \\
&\cdot \frac{1}{[1 + \rho(|g_1|^2 + |g_2|^2)]^T} \\
&\cdot \frac{1}{[1 + \rho(|g_1|^2 + |g_2|^2 + |g_r|^2)]^{(M-m)T}}
\end{aligned} \tag{61}$$

Notice that (61) does not depend on m' and follows from the inequality

$$\det\left(\mathbf{I} + \frac{k\rho\mathbf{C}^H\mathbf{C}}{2}\right) \geq \left[1 + \frac{k\rho}{2}(|h_1|^2 + |h_2|^2 + |g_1|^2 + |g_2|^2)\right]$$

Define $|h_i|^2 = \rho^{-\alpha_i}$, $|g_i|^2 = \rho^{-\beta_i}$. Use the union bound for $\overline{\mathcal{B}}_m$, summing over all $m' = 1, \dots, M$ and over all messages $\{\tilde{w}_1 \neq 1, \tilde{w}_2 \neq 1\}$, and average over the channel realizations where $\{(h_1, h_2) \notin \mathcal{O}_R^{m-1}\}$, $\{(g_1, g_2, g_r) \notin \mathcal{O}_D^m\}$, use the techniques developed in [3], we obtain the exponent of this

correlated term as

$$\begin{aligned} d_{m,cor}^{s=0}(r) &= \inf_{\substack{\alpha_i, \beta_i \geq 0, \\ \mathcal{O}_R^{m-1} \cap \mathcal{O}_D^m}} \{ \alpha_1 + \alpha_2 + \beta_1 + \beta_2 + \beta_r + T f_m^{s=0}(\alpha_i, \beta_i, r) \} \\ &\geq \inf_{\substack{\alpha_i, \beta_i \geq 0, \\ \mathcal{O}_R^{m-1}}} \{ \alpha_1 + \alpha_2 + \beta_1 + \beta_2 + \beta_r + T f_m^{s=0}(\alpha_i, \beta_i, r) \} \end{aligned} \quad (62)$$

$$\geq \inf_{\substack{\alpha_i, \beta_i \geq 0, \\ \mathcal{O}_R^{m-1}}} \{ \alpha_1 + \alpha_2 + T f_m^{s=0}(\alpha_i, \beta_i, r) \} \quad (63)$$

$$+ \inf_{\substack{\alpha_i, \beta_i \geq 0, \\ \mathcal{O}_R^{m-1}}} \{ \beta_1 + \beta_2 + \beta_r + T f_m^{s=0}(\alpha_i, \beta_i, r) \} \quad (64)$$

$$\geq d_{m,R}(r) + d_{m,D}(r) \quad (65)$$

where

$$\begin{aligned} f_m^{s=0}(\alpha_i, \beta_i, r) &= (m-1) \max[(1-\alpha_1)^+, (1-\alpha_2)^+, (1-\beta_1)^+, (1-\beta_2)^+] \\ &\quad + \max[(1-\beta_1)^+, (1-\beta_2)^+] \\ &\quad + (M-m) \max[(1-\beta_1)^+, (1-\beta_2)^+, (1-\beta_r)^+] - rM \end{aligned} \quad (66)$$

(63), (64) follows from the fact that the infimum of (62) occurs where $f_m^{s=0}(\alpha_i, \beta_i, r) \rightarrow 0$ if T is large enough (i.e. $T \geq 4$). Notice the event

$$\{(\alpha_i) : (m-1) \max[(1-\alpha_1)^+, (1-\alpha_2)^+] - rM > 0\} \in \overline{\mathcal{O}_R^{m-1}} \quad (67)$$

It can be checked that (63) $\geq d_{m,R}(r)$, and (64) $\geq d_{m,D}(r)$.

(II) For $\{\tilde{w}_1 \neq 1, \tilde{w}_2 = 1\}$, $s = 2$,

$$\begin{aligned} \frac{1}{\det(\mathbf{I} + \frac{k\mathbf{DR}^s}{4})} &\doteq \frac{1}{[1 + \rho(|h_1|^2 + |g_1|^2)]^{(m-1)T}} \\ &\quad \cdot \frac{1}{[1 + \rho|g_1|^2]^T} \\ &\quad \cdot \frac{1}{[1 + \rho(|g_1|^2 + |g_r|^2)]^{(M-m)T}} \end{aligned} \quad (68)$$

Apply similar arguments, we obtain the exponent of this correlated term as

$$\begin{aligned} d_{m,cor}^{s=2}(r) &= \inf_{\substack{\alpha_i, \beta_i \geq 0, \\ \mathcal{O}_R^{m-1} \cap \mathcal{O}_D^m}} \{ \alpha_1 + \beta_1 + \beta_r + T f_m^{s=2}(\alpha_i, \beta_i, r) \} \\ &\geq \inf_{\substack{\alpha_i, \beta_i \geq 0, \\ \mathcal{O}_R^{m-1}}} \{ \alpha_1 + \beta_1 + \beta_r + T f_m^{s=2}(\alpha_i, \beta_i, r) \} \end{aligned} \quad (69)$$

$$\geq \inf_{\substack{\alpha_i, \beta_i \geq 0, \\ \mathcal{O}_R^{m-1}}} \{ \alpha_1 + T f_m^{s=2}(\alpha_i, \beta_i, r) \} \quad (70)$$

$$+ \inf_{\substack{\alpha_i, \beta_i \geq 0, \\ \mathcal{O}_R^{m-1}}} \{ \beta_1 + \beta_r + T f_m^{s=2}(\alpha_i, \beta_i, r) \} \quad (71)$$

$$\geq d_{m,R}(r) + d_{m,D}(r) \quad (72)$$

where

$$\begin{aligned} f_m^{s=2}(\alpha_i, \beta_i, r) &= (m-1) \max[(1-\alpha_1)^+, (1-\beta_1)^+] \\ &\quad + \max(1-\beta_1)^+ \\ &\quad + (M-m) \max[(1-\beta_1)^+, (1-\beta_r)^+] - \frac{r}{2}M \end{aligned} \quad (73)$$

Note

$$\left\{ \alpha_1 : (m-1) \max(1-\alpha_1)^+ - \frac{r}{2}M > 0 \right\} \in \overline{\mathcal{O}_R^{m-1}} \quad (74)$$

The case for $\{\tilde{w}_1 = 1, \tilde{w}_2 \neq 1\}$, $s = 1$ is similar and omitted here. Thus this correlated term can be ignored with respect to the term (I) in DMT analysis.

For averaging the first sum in (58), define

$$\mathbf{H}_i^{s'} = \begin{cases} \mathbf{h}'^H \mathbf{h}' & 1 \leq i \leq (m-1)T \\ \mathbf{0} & \text{otherwise} \end{cases}$$

where

$$\mathbf{h}' = \begin{bmatrix} h'_1 & h'_2 & 0 & 0 \end{bmatrix}$$

and s is chosen according to w as before, we have

$$\left(\sum_{w \neq \{(1,1), \tilde{w}\}} \frac{1}{\det(\mathbf{I} + \frac{k\mathbf{D}\mathbf{H}^s}{4})} \right) \quad (75)$$

Averaged by $\{(h_1, h_2) \notin \mathcal{O}_R^{m-1}\}$, $\{(g_1, g_2, g_r) \notin \mathcal{O}_D^m\}$, the exponent $d_{m,(h_i)}(r)$ can be derived.

To average the second sum in (58), let $\mathbf{Q}^s = \mathbf{R}^s$, except for $1 \leq i \leq (m-1)T$, $\mathbf{Q}_i^s = \mathbf{R}_{mT}^s$, use the union bound over \tilde{w} , by averaging the term

$$\left(\sum_{\tilde{w} \neq (1,1)} \frac{1}{\det \left(\mathbf{I} + \frac{k\mathbf{DQ}^s}{4} \right)} \right) \quad (76)$$

over $\{(h_1, h_2) \notin O_R^{m-1}\}$, $\{(g_1, g_2, g_r) \notin O_D^m\}$, we obtain the exponent $d_{m,(g_i)}(r)$. Analogous to Appendix A-A, Appendix A-B, it can be shown that $d_{m,(h_i)}(r) \geq d_{m,R}(r)$, $d_{m,(g_i)}(r) \doteq d_{m,D}(r)$. Collecting all the results above, the term (2) can be ignored compared to the term (1) in DMT analysis.

(3) $P(\{(g_1, g_2, g_r) \in O_D^m\})P(\{(h_1, h_2) \notin O_R^{m-1}\}, \{\mathbf{y}_{r,0}^{m-1} \notin \mathcal{U}_{m-1}\})$: These two terms can be averaged over the Gaussian random ensemble separately. It is clear the first term has the exponent of $d_{m,D}(r)$, and the second has $d_{m,(h_i)}(r)$.

(4) $P(E, \{(g_1, g_2, g_r) \notin O_D^m\} | \bar{E}_r, \mathbf{m} = m)P(\{(h_1, h_2) \in O_R^{m-1}\}, \{(h_1, h_2) \notin O_R^m\})$: These two terms can be averaged over the Gaussian random ensemble separately. It is clear the first term has the exponent of $d_{m,(g_i)}(r)$, and the second has $d_{m,R}(r)$.

APPENDIX C

PROOF OF THEOREM 3

To upper bound the second term on the right hand side of (26), we further partition the error event into two error events depending on the MAF's outage event O_{MAF} of α_i, β_i , where the MAF is being used,

$$\begin{aligned} & P \left(E, \left\{ \{(h_1, h_2) \notin O_R^{\frac{M}{2}}\}, \{\mathbf{y}_{r,0}^{\frac{M}{2}} \notin \mathcal{U}_{\frac{M}{2}}\} \right\} \right) = \\ & P \left(E, O_{MAF}, \left\{ \{(h_1, h_2) \notin O_R^{\frac{M}{2}}\}, \{\mathbf{y}_{r,0}^{\frac{M}{2}} \notin \mathcal{U}_{\frac{M}{2}}\} \right\} \right) + \\ & P \left(E, \bar{O}_{MAF}, \left\{ \{(h_1, h_2) \notin O_R^{\frac{M}{2}}\}, \{\mathbf{y}_{r,0}^{\frac{M}{2}} \notin \mathcal{U}_{\frac{M}{2}}\} \right\} \right) \end{aligned} \quad (77)$$

Notice we do not apply the standard DMT analysis [9] here by simply using $P(O_{MAF})$ to upper bound the first term on the right hand side of (77) (denoted as $P^{1st}(\xi)$ for notation ease), otherwise the lower bound of DMT we obtain for (77) will be the same as MAF protocol, only $2 - \frac{3r}{2}$, not $2 - r$.

For the the analysis of P_ξ^{1st} , note the fact

$$P\left(E \left| (\alpha_i, \beta_i) \in O_{MAF}, \left\{ \{(h_1, h_2) \notin O_R^{\frac{M}{2}}\}, \{\mathbf{y}_{r,0}^{\frac{M}{2}} \notin \mathcal{U}_{\frac{M}{2}}\} \right\} \right) \leq 1\right.$$

we have

$$\begin{aligned} & P^{1st}(\xi | (\alpha_i, \beta_i) \in O_{MAF}) \\ & \leq P\left(\left\{ \{(h_1, h_2) \notin O_R^{\frac{M}{2}}\}, \{\mathbf{y}_{r,0}^{\frac{M}{2}} \notin \mathcal{U}_{\frac{M}{2}}\} \right\} \left| (\alpha_i, \beta_i) \in O_{MAF} \right.\right) \end{aligned}$$

Recall (49), (50), by averaging over the Gaussian random codebook, we have

$$\begin{aligned} & P^{1st}(\xi | (\alpha_i, \beta_i) \in O_{MAF}) \\ & \leq \sum_{i=1}^2 \rho^{-\frac{MT}{2}[(1-\alpha_i)-\frac{r}{2}]} + \rho^{-\frac{MT}{2}[(1-\min(\alpha_1, \alpha_2))-r]} \end{aligned} \quad (78)$$

Finally, averaging the right hand side of (78) over $(\alpha_i, \beta_i) \in \left\{ O_{MAF} \cap \left((\alpha_1, \alpha_2) \notin O_R^{\frac{M}{2}} \right) \right\}$, for T is large enough (but finite), the derivation of $P^{1st}(\xi) \doteq \rho^{d_{P^{1st}}(\xi)}$ becomes identical to (29), (31) and $d_{P^{1st}}(\xi) \geq 2 - r$. Then we turn to upper bound the second term on the right hand side of (77), denoted as $P^{2nd}(\xi)$. As (25), $P^{2nd}(\xi) \leq \sum_I P^{2nd}(\xi_I)$. Use $P_{MAF}(E, \overline{O}_{MAF})$, where MAF is always used, to upper bound $P^{2nd}(\xi)$, we have $P^{2nd}(\xi_{(1,2)}) \geq 3 - r$, however, only $P^{2nd}(\xi_i) \geq 2 - \frac{3r}{2}$ is obtained, $i = 1, 2$. To get a tighter upper bound for $P^{2nd}(\xi_1)$, (49), (50) are again used and apply the techniques developed in Appendix B-C to MAF protocol, where the equivalent MAF model can be expressed as

$$\mathbf{y} = \begin{bmatrix} g_1 \mathbf{I}_{MT/2} & \mathbf{0} \\ g_2 h_1 b \mathbf{I}_{MT/2} & g_1 \mathbf{I}_{MT/2} \end{bmatrix} \mathbf{x}_1 + \begin{bmatrix} \mathbf{0} \\ g_2 b \mathbf{I}_{MT/2} \end{bmatrix} \mathbf{n} + \mathbf{v} \quad (79)$$

and b is chosen to be of exponential order zero [6], [3], which satisfies the relay's transmission power constraint. We have

$$\begin{aligned} & P_1^{2nd}(\{(1, 1) \rightarrow \tilde{w}\}) \leq e^{-|\mathbf{z}'|^2} + \\ & \sum_{w \neq \{(1, 1), \tilde{w}\}} \exp \left\{ -\frac{|h_1 \Delta \mathbf{x}_{1,0}^{\frac{M}{2}}(w_1) + h_2 \Delta \mathbf{x}_{2,0}^{\frac{M}{2}}(w_2)|^2}{(2MT(1+\delta)\sigma_n^2)} \right\} e^{-|\mathbf{z}|^2} \end{aligned} \quad (80)$$

where $P_1^{2nd}(\{(1, 1) \rightarrow \tilde{w}\})$ denotes the probability that $(1, 1)$ is decoded as \tilde{w} for type-1 error in the event of the second term in (77) given h_i, g_i and \mathbf{z} and \mathbf{z}' are redefined in this section as

$$\mathbf{z} = \begin{bmatrix} \frac{g_1}{\sigma_n} \mathbf{I}_{MT/2} & \mathbf{0} \\ \frac{g_2 h_1 b}{\sqrt{\sigma_n^2 + |g_2 b|^2} \sigma_v} \mathbf{I}_{MT/2} & \frac{g_1}{\sqrt{\sigma_n^2 + |g_2 b|^2} \sigma_v} \mathbf{I}_{MT/2} \end{bmatrix} \Delta \mathbf{x}_1(\tilde{w}) \quad (81)$$

$$\mathbf{z}' = \begin{bmatrix} \frac{h_1}{\sigma_n^2} \mathbf{I}_{MT/2} & \mathbf{0} \\ \frac{g_1}{\sigma_n^2} \mathbf{I}_{MT/2} & \mathbf{0} \\ \frac{g_2 h_1 b}{\sqrt{\sigma_n^2 + |g_2 b|^2} \sigma_v^2} \mathbf{I}_{MT/2} & \frac{g_1}{\sqrt{\sigma_n^2 + |g_2 b|^2} \sigma_v^2} \mathbf{I}_{MT/2} \end{bmatrix} \Delta \mathbf{x}_1(\tilde{w}) \quad (82)$$

Apply the union bound and average them over Gaussian random ensemble and corresponding channel realizations. For the product term on the right hand side of (80), it can be averaged separately over the Gaussian random ensemble, resulting in the exponent $d_{product}(r)$ identical to (29). For the first term on the right hand side of (80), we obtain its exponent equal to

$$\inf_{(\alpha_i, \beta_i) \in C} \left\{ \alpha_1 + \alpha_2 + \beta_1 + \beta_2 + \beta_r + \frac{MT}{2} k(\alpha_i, \beta_i, r) \right\} \geq 2 - r \quad (83)$$

where

$$k(\alpha_i, \beta_i, r) = \max \{ 2(1 - \beta_1), 1 - (\beta_r + \alpha_1), 2 - (\alpha_1 + \beta_1) \} - r$$

$$\text{and } C = \left\{ \overline{O}_{MAF} \cap \left((\alpha_1, \alpha_2) \notin O_R^{\frac{M}{2}} \right) \right\}$$

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